

SI UNITS USED



SIXTH EDITION

ELECTRICAL AND
ELECTRONIC PRINCIPLES
AND TECHNOLOGY

JOHN BIRD

Electrical and Electronic Principles and Technology

This practical resource introduces electrical and electronic principles and technology covering theory through detailed examples, enabling students to develop a sound understanding of the knowledge required by technicians in fields such as electrical engineering, electronics and telecommunications. No previous background in engineering is assumed, making this an ideal text for vocational courses at Levels 2 and 3, foundation degrees and introductory courses for undergraduates.

John Bird, BSc (Hons), CEng, CSci, CMath, FITE, FIMA, FCollT, is the former Head of Applied Electronics in the Faculty of Technology at Highbury College, Portsmouth, UK. More recently he has combined freelance lecturing and examining, and is the author of over 130 textbooks on engineering and mathematical subjects with worldwide sales of over one million copies. He is currently lecturing at the Defence School of Marine and Air Engineering in the Defence College of Technical Training at HMS Sultan, Gosport, Hampshire, UK.

To Sue

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Sixth edition

John Bird

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Preface

Electrical and Electronic Principles and Technology, 6th Edition introduces the principles which describe the operation of d.c. and a.c. circuits, covering both steady and transient states, and applies these principles to filter networks, operational amplifiers, three-phase supplies, transformers, d.c. machines and three-phase induction motors.

In this new sixth edition, **new material added** includes some mathematics revision needed for electrical and electronic principles, ways of generating electricity – the present and the future (including more on renewable energy), more on lithium-ion batteries and solar energy, along with other minor modifications.

This sixth edition of the textbook provides coverage of the following latest syllabuses:

- (i) ‘**Electrical and Electronic Principles**’ (BTEC Level 3 Nationals Specification in Engineering, Unit 6) – see chapters 3–12, 13 (part), 14 (part), 16, 17 (part), 18 (part), 20 (part), 24 (part), 25 (part).
- (ii) ‘**Further Electrical Principles**’ (BTEC National Certificate and National Diploma, Unit 64) – see chapters 15, 17–20, 23, 25, 26.
- (iii) Parts of the following BTEC National syllabuses: **Electrical Technology** (Unit 51), **Principles and operation of three-phase systems** (Unit 62) and **Three-phase motors and drives** (Unit 63).
- (iv) Various parts of City & Guilds Technician Certificate/Diploma in Electrical and Electronic Engineering/Telecommunication Systems.
- (v) ‘Electrical and Electronic Principles’ (EAL Advanced Diploma in Engineering and Technology).
- (vi) Any introductory/Access/Foundation course involving Electrical and Electronic Engineering Principles.

The **text** is set out in five main sections:

Part 1, comprising chapters 1 and 2, involves **Revision of Some Basic Mathematics** needed for Electrical and Electronic Principles.

Part 2, comprising chapters 3 to 14, involves essential **Basic Electrical and Electronic Engineering Principles**, with chapters on electrical units and quantities, introduction to electric circuits, resistance variation, chemical effects of electricity, series and parallel networks, capacitors and capacitance, magnetic circuits, electromagnetism, electromagnetic induction, electrical measuring instruments and measurements, semiconductor diodes and transistors.

Part 3, comprising chapters 15 to 21, involves **Further Electrical and Electronic Principles**, with chapters on d.c. circuit theory, alternating voltages and currents, single-phase series and parallel networks, filter networks, d.c. transients and operational amplifiers.

Part 4, comprising chapters 22 to 26, involves **Electrical Power Technology**, with chapters on ways of generating electricity – the present and the future, three-phase systems, transformers, d.c. machines and three-phase induction motors.

Part 5, comprising chapter 27, details **10 practical laboratory experiments**.

Each topic considered in the text is presented in a way that assumes in the reader little previous knowledge of that topic. Theory is introduced in each chapter by a reasonably brief outline of essential information, definitions, formulae, procedures, etc. The theory is kept to a minimum, for problem solving is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

Electrical and Electronic Principles and Technology, 6th Edition contains some **500 worked problems** to

aid understanding. Also included are over **500 short answer questions**, the answers for which can be determined from the preceding material in that particular chapter, together with **404 multi-choice questions**, and over **850 further questions**, arranged in **172 Practice Exercises**, all with answers at the back of the book. The Practice Exercises appear at regular intervals – every 3 or 4 pages – throughout the text. **622 line diagrams** further enhance the understanding of the theory. All of the problems – multi-choice, short answer and further questions – mirror practical situations found in electrical and electronic engineering.

At regular intervals throughout the text are seven **Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in chapters 3 to 6, Revision Test 2 covers the material contained in chapters 7 to 9, and so on. These Revision Tests do not have answers given since it is envisaged that lecturers/instructors could set the Tests for students to attempt as part of their course structure. Lecturers/instructors may access a free Internet download of full solutions of the Revision Tests in an **Instructor's Manual** – see below.

I am once again grateful to **Mike Tooley** for his help and advice in the work involved in preparing this sixth edition of the text

A list of relevant **formulae** are included at the end of each of the first four parts of the book.

'**Learning by Example**' is at the heart of *Electrical and Electronic Principles and Technology, 6th Edition*.

JOHN BIRD
**Royal Naval Defence College of Marine
 and Air Engineering, HMS Sultan,
 formerly University of Portsmouth
 and Highbury College, Portsmouth**

John Bird is the former Head of Applied Electronics in the Faculty of Technology at Highbury College, Portsmouth, UK. More recently, he has combined freelance lecturing at the University of Portsmouth, with examiner responsibilities for Advanced Mathematics with City and Guilds, and examining for International Baccalaureat. He is the author of some 130 textbooks on engineering and mathematical subjects with worldwide sales of over one million copies. He is currently lecturing at the Defence School of Marine and Air Engineering in the Defence College of Technical Training at HMS Sultan, Gosport, Hampshire, UK.

Free Web downloads

The following support material is available from www.routledge.com/cw/bird

For Students:

1. **Full solutions to all 850 further questions in the Practice Exercises**
2. **A set of formulae for each of the first four parts of the text**
3. **Multiple choice questions sheets**
4. **Information on 29 Engineers/Scientists mentioned in the text**

For Lecturers/Instructors:

1. **Full solutions to all 850 further questions in the Practice Exercises**
2. **Full solutions and marking scheme for each of the 7 Revision Tests. Also, each test may be downloaded.**
3. **Lesson Plans and revision material.** Typical 30-week lesson plans for 'Electrical and Electronic Principles', Unit 6, and 'Further Electrical Principles', Unit 64, are included, together with two practice examination question papers (with solutions) for each of the modules.
4. **Laboratory Experiments.** In chapter 27, 10 practical laboratory experiments are included. It maybe that tutors will want to edit these experiments to suit their own equipment/component availability.
5. **A set of formulae for each of the first four parts of the text**
6. **Multiple choice questions sheets**
7. **Information on 29 Engineers/Scientists mentioned in the text**
8. **All 622 illustrations used in the text may be downloaded for use in PowerPoint presentations**



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Revision of Some Basic Mathematics



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Some mathematics revision

Why it is important to understand: Some mathematics revision

Mathematics is a vital tool for professional and chartered engineers. It is used in electrical and electronic engineering, in mechanical and manufacturing engineering, in civil and structural engineering, in naval architecture and marine engineering and in aeronautical and rocket engineering. In these various branches of engineering, it is very often much cheaper and safer to design your artefact with the aid of mathematics – rather than through guesswork. ‘Guesswork’ may be reasonably satisfactory if you are designing an exactly similar artefact as one that has already proven satisfactory; however, the classification societies will usually require you to provide the calculations proving that the artefact is safe and sound. Moreover, these calculations may not be readily available to you and you may have to provide fresh calculations, to prove that your artefact is ‘roadworthy’. For example, if you design a tall building or a long bridge by ‘guesswork’, and the building or bridge do not prove to be structurally reliable, it could cost you a fortune to rectify the deficiencies. This cost may dwarf the initial estimate you made to construct these structures, and cause you to go bankrupt. Thus, without mathematics, the prospective professional or chartered engineer is very severely disadvantaged.

Knowledge of mathematics provides the basis for all engineering.

At the end of this chapter you should be able to:

- use a calculator and evaluate formulae
- manipulate fractions
- understand and perform calculations with percentages
- appreciate ratios and direct and inverse proportion
- understand and use the laws of indices
- expand equations containing brackets
- solve simple equations
- transpose formulae
- solve simultaneous equations in two unknowns

1.1 Use of calculator and evaluating formulae

In engineering, calculations often need to be performed. For simple numbers it is useful to be able to use

mental arithmetic. However, when numbers are larger an electronic calculator needs to be used.

In engineering calculations it is essential to have a **scientific notation calculator** which will have all the necessary functions needed, and more. This chapter assumes you have a **CASIO fx-991ES PLUS**

calculator, or similar. If you can accurately use a calculator, your confidence with engineering calculations will improve.

Check that you can use a calculator in the following Practice Exercise

**Practice Exercise 1 Use of calculator
(Answers on page 504)**

- Evaluate $378.37 - 298.651 + 45.64 - 94.562$
- Evaluate $\frac{17.35 \times 34.27}{41.53 \div 3.76}$ correct to 3 decimal places
- Evaluate $\frac{(4.527 + 3.63)}{(452.51 \div 34.75)} + 0.468$ correct to 5 significant figures
- Evaluate $52.34 - \frac{(912.5 \div 41.46)}{(24.6 - 13.652)}$ correct to 3 decimal places
- Evaluate $\frac{52.14 \times 0.347 \times 11.23}{19.73 \div 3.54}$ correct to 4 significant figures
- Evaluate 6.85^2 correct to 3 decimal places
- Evaluate $(0.036)^2$ in engineering form
- Evaluate 1.3^3
- Evaluate $(0.38)^3$ correct to 4 decimal places
- Evaluate $(0.018)^3$ in engineering form
- Evaluate $\frac{1}{0.00725}$ correct to 1 decimal place
- Evaluate $\frac{1}{0.065} - \frac{1}{2.341}$ correct to 4 significant figures
- Evaluate 2.1^4
- Evaluate $(0.22)^5$ correct to 5 significant figures in engineering form
- Evaluate $(1.012)^7$ correct to 4 decimal places
- Evaluate $1.1^3 + 2.9^4 - 4.4^2$ correct to 4 significant figures
- Evaluate $\sqrt{34528}$ correct to 2 decimal places
- Evaluate $\sqrt[3]{17}$ correct to 3 decimal places
- Evaluate $\sqrt[6]{2451} - \sqrt[4]{46}$ correct to 3 decimal places

Express the answers to questions 20 to 23 in engineering form.

- Evaluate $5 \times 10^{-3} \times 7 \times 10^8$
- Evaluate $\frac{6 \times 10^3 \times 14 \times 10^{-4}}{2 \times 10^6}$

22. Evaluate $\frac{56.43 \times 10^{-3} \times 3 \times 10^4}{8.349 \times 10^3}$ correct to 3 decimal places

23. Evaluate $\frac{99 \times 10^5 \times 6.7 \times 10^{-3}}{36.2 \times 10^{-4}}$ correct to 4 significant figures

24. Evaluate $\frac{4}{5} - \frac{1}{3}$ as a decimal, correct to 4 decimal places

25. Evaluate $\frac{2}{3} - \frac{1}{6} + \frac{3}{7}$ as a fraction

26. Evaluate $2\frac{5}{6} + 1\frac{5}{8}$ as a decimal, correct to 4 significant figures

27. Evaluate $5\frac{6}{7} - 3\frac{1}{8}$ as a decimal, correct to 4 significant figures

28. Evaluate $\frac{3}{4} \times \frac{4}{5} - \frac{2}{3} \div \frac{4}{9}$ as a fraction

29. Evaluate $8\frac{8}{9} \div 2\frac{2}{3}$ as a mixed number

30. Evaluate $3\frac{1}{5} \times 1\frac{1}{3} - 1\frac{7}{10}$ as a decimal, correct to 3 decimal places

31. Evaluate $\frac{\left(4\frac{1}{5} - 1\frac{2}{3}\right)}{\left(3\frac{1}{4} \times 2\frac{3}{5}\right)} - \frac{2}{9}$ as a decimal, correct to 3 significant figures

In questions 32 to 38, evaluate correct to 4 decimal places.

32. Evaluate $\sin 67^\circ$

33. Evaluate $\tan 71^\circ$

34. Evaluate $\cos 63.74^\circ$

35. Evaluate $\tan 39.55^\circ - \sin 52.53^\circ$

36. Evaluate $\sin(0.437 \text{ rad})$

37. Evaluate $\tan(5.673 \text{ rad})$

38. Evaluate $\frac{(\sin 42.6^\circ)(\tan 83.2^\circ)}{\cos 13.8^\circ}$

In questions 39 to 45, evaluate correct to 4 significant figures.

39. 1.59π 40. $2.7(\pi - 1)$

41. $\pi^2(\sqrt{13} - 1)$ 42. $8.5e^{-2.5}$

43. $3e^{(2\pi-1)}$

44. $\sqrt{\left[\frac{5.52\pi}{2e^{-2} \times \sqrt{26.73}}\right]}$

45. $\sqrt{\left[\frac{e^{(2-\sqrt{3})}}{\pi \times \sqrt{8.57}}\right]}$

Evaluation of formulae

The statement $y = mx + c$ is called a **formula** for y in terms of m , x and c ; m , x and c are called **symbols**.

When given values of m , x and c we can evaluate y .

There are a large number of formulae used in engineering and in this section we will insert numbers in place of symbols to evaluate engineering quantities.

Here are some practical examples. Check with your calculator that you agree with the working and answers.

Problem 1. In an electrical circuit the voltage V is given by Ohm's law, i.e. $V = IR$. Find, correct to 4 significant figures, the voltage when $I = 5.36$ A and $R = 14.76$ Ω

$$V = IR = I \times R = 5.36 \times 14.76$$

Hence, **voltage $V = 79.11$ V, correct to 4 significant figures**

Problem 2. Velocity v is given by $v = u + at$. If $u = 9.54$ m/s, $a = 3.67$ m/s² and $t = 7.82$ s, find v , correct to 3 significant figures.

$$\begin{aligned} v &= u + at = 9.54 + 3.67 \times 7.82 \\ &= 9.54 + 28.6994 = 38.2394 \end{aligned}$$

Hence, **velocity $v = 38.2$ m/s, correct to 3 significant figures**

Problem 3. The area, A , of a circle is given by $A = \pi r^2$. Determine the area correct to 2 decimal places, given radius $r = 5.23$ m

$$A = \pi r^2 = \pi (5.23)^2 = \pi (27.3529)$$

Hence, **area, $A = 85.93$ m², correct to 2 decimal places**

Problem 4. Density = $\frac{\text{mass}}{\text{volume}}$. Find the density when the mass is 6.45 kg and the volume is 300×10^{-6} m³.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{6.45 \text{ kg}}{300 \times 10^{-6} \text{ m}^3} = 21500 \text{ kg/m}^3$$

Problem 5. The power, P watts, dissipated in an electrical circuit is given by the formula $P = \frac{V^2}{R}$. Evaluate the power, correct to 4 significant figures, given that $V = 230$ V and $R = 35.63$ Ω

$$P = \frac{V^2}{R} = \frac{(230)^2}{35.63} = \frac{52900}{35.63} = 1484.70390\dots$$

Press ENG and 1.48470390.. $\times 10^3$ appears on the screen.

Hence, **power, $P = 1485$ W or 1.485 kW correct to 4 significant figures.**

Problem 6. Resistance, R Ω , varies with temperature according to the formula $R = R_0(1 + \alpha t)$. Evaluate R , correct to 3 significant figures, given $R_0 = 14.59$, $\alpha = 0.0043$ and $t = 80$.

$$\begin{aligned} R &= R_0(1 + \alpha t) = 14.59[1 + (0.0043)(80)] \\ &= 14.59(1 + 0.344) = 14.59(1.344) \end{aligned}$$

Hence, **resistance, $R = 19.6$ Ω , correct to 3 significant figures**

Problem 7. The current, I amperes, in an a.c. circuit is given by: $I = \frac{V}{\sqrt{(R^2 + X^2)}}$

Evaluate the current, correct to 2 decimal places, when $V = 250$ V, $R = 25.0$ Ω and $X = 18.0$ Ω .

$$I = \frac{V}{\sqrt{(R^2 + X^2)}} = \frac{250}{\sqrt{(25.0^2 + 18.0^2)}} = 8.11534341\dots$$

Hence, **current, $I = 8.12$ A, correct to 2 decimal places**

Now try the following Practice Exercise

Practice Exercise 2 Evaluation of formulae (Answers on page 504)

- The area A of a rectangle is given by the formula $A = l \times b$. Evaluate the area, correct to 2 decimal places, when $l = 12.4$ cm and $b = 5.37$ cm.

2. The circumference C of a circle is given by the formula $C = 2\pi r$. Determine the circumference, correct to 2 decimal places, given $r = 8.40$ mm.
3. A formula used in connection with gases is $R = \frac{PV}{T}$. Evaluate R when $P = 1500$, $V = 5$ and $T = 200$.
4. The velocity of a body is given by $v = u + at$. The initial velocity u is measured when time t is 15 seconds and found to be 12 m/s. If the acceleration a is 9.81 m/s^2 calculate the final velocity v .
5. Calculate the current I in an electrical circuit, correct to 3 significant figures, when $I = V/R$ amperes when the voltage V is measured and found to be 7.2 V and the resistance R is 17.7Ω .
6. Find the distance s , given that $s = \frac{1}{2}gt^2$. Time $t = 0.032$ seconds and acceleration due to gravity $g = 9.81 \text{ m/s}^2$. Give the answer in millimetres correct to 3 significant figures.
7. The energy stored in a capacitor is given by $E = \frac{1}{2}CV^2$ joules. Determine the energy when capacitance $C = 5 \times 10^{-6}$ farads and voltage $V = 240$ V.
8. Find the area A of a triangle, correct to 1 decimal place, given $A = \frac{1}{2}bh$, when the base length b is 23.42 m and the height h is 53.7 m.
9. Resistance R_2 is given by $R_2 = R_1(1 + \alpha t)$. Find R_2 , correct to 4 significant figures, when $R_1 = 220$, $\alpha = 0.00027$ and $t = 75.6$.
10. Density = $\frac{\text{mass}}{\text{volume}}$. Find the density, correct to 4 significant figures, when the mass is 2.462 kg and the volume is 173 cm^3 . Give the answer in units of kg/m^3 . Note that $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$.
11. Evaluate resistance R_T , correct to 4 significant figures, given $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ when $R_1 = 5.5 \Omega$, $R_2 = 7.42 \Omega$ and $R_3 = 12.6 \Omega$.
12. The potential difference, V volts, available at battery terminals is given by $V = E - Ir$. Evaluate V when $E = 5.62$, $I = 0.70$ and $R = 4.30$.
13. The current I amperes flowing in a number of cells is given by $I = \frac{nE}{R + nr}$. Evaluate the current, correct to 3 significant figures, when $n = 36$, $E = 2.20$, $R = 2.80$ and $r = 0.50$.
14. Energy, E joules, is given by the formula $E = \frac{1}{2}LI^2$. Evaluate the energy when $L = 5.5$ H and $I = 1.2$ A.
15. The current I amperes in an a.c. circuit is given by $I = \frac{V}{\sqrt{R^2 + X^2}}$. Evaluate the current, correct to 4 significant figures, when $V = 250$ V, $R = 11.0 \Omega$ and $X = 16.2 \Omega$.

1.2 Fractions

An example of a fraction is $\frac{2}{3}$ where the top line, i.e. the 2, is referred to as the **numerator** and the bottom line, i.e. the 3, is referred to as the **denominator**.

A **proper fraction** is one where the numerator is smaller than the denominator, examples being $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$, and so on.

An **improper fraction** is one where the denominator is smaller than the numerator, examples being $\frac{3}{2}$, $\frac{2}{1}$, $\frac{8}{3}$, $\frac{16}{5}$, and so on.

Addition of fractions is demonstrated in the following worked problems.

Problem 8. Evaluate A , given $A = \frac{1}{2} + \frac{1}{3}$

The lowest common denominator of the two denominators 2 and 3 is 6, i.e. 6 is the lowest number that both 2 and 3 will divide into.

Then $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$ i.e. both $\frac{1}{2}$ and $\frac{1}{3}$ have the common denominator, namely 6.

The two fractions can therefore be added as:

$$A = \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$$

Problem 9. Evaluate A, given $A = \frac{2}{3} + \frac{3}{4}$

A common denominator can be obtained by multiplying the two denominators together, i.e. the common denominator is $3 \times 4 = 12$.

The two fractions can now be made equivalent,

$$\text{i.e. } \frac{2}{3} = \frac{8}{12} \text{ and } \frac{3}{4} = \frac{9}{12}$$

so that they can be easily added together, as follows:

$$A = \frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{8+9}{12} = \frac{17}{12}$$

$$\text{i.e. } A = \frac{2}{3} + \frac{3}{4} = 1\frac{5}{12}$$

Problem 10. Evaluate A, given $A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2}$

A suitable common denominator can be obtained by multiplying $6 \times 7 = 42$, and all three denominators divide exactly into 42.

$$\text{Thus, } \frac{1}{6} = \frac{7}{42}, \frac{2}{7} = \frac{12}{42} \text{ and } \frac{3}{2} = \frac{63}{42}$$

$$\text{Hence, } A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2} = \frac{7}{42} + \frac{12}{42} + \frac{63}{42}$$

$$= \frac{7+12+63}{42} = \frac{82}{42} = \frac{41}{21}$$

$$\text{i.e. } A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2} = 1\frac{20}{21}$$

Problem 11. Determine A as a single fraction, given $A = \frac{1}{x} + \frac{2}{y}$

A common denominator can be obtained by multiplying the two denominators together, i.e. xy

$$\text{Thus, } \frac{1}{x} = \frac{y}{xy} \text{ and } \frac{2}{y} = \frac{2x}{xy}$$

$$\text{Hence, } A = \frac{1}{x} + \frac{2}{y} = \frac{y}{xy} + \frac{2x}{xy} \quad \text{i.e. } A = \frac{y+2x}{xy}$$

Note that addition, subtraction, multiplication and division of fractions may be determined using a **calculator** (for example, the CASIO fx-991ES PLUS).

Locate the $\frac{\square}{\square}$ and $\square \frac{\square}{\square}$ functions on your calculator (the latter function is a shift function found above the $\frac{\square}{\square}$ function) and then check the following worked problems.

Problem 12. Evaluate $\frac{1}{4} + \frac{2}{3}$ using a calculator

- (i) Press $\frac{\square}{\square}$ function
- (ii) Type in 1
- (iii) Press \downarrow on the cursor key and type in 4
- (iv) $\frac{1}{4}$ appears on the screen
- (v) Press \rightarrow on the cursor key and type in +
- (vi) Press $\frac{\square}{\square}$ function
- (vii) Type in 2
- (viii) Press \downarrow on the cursor key and type in 3
- (ix) Press \rightarrow on the cursor key
- (x) Press = and the answer $\frac{11}{12}$ appears
- (xi) Press S \Leftrightarrow D function and the fraction changes to a decimal 0.9166666...

Thus, $\frac{1}{4} + \frac{2}{3} = \frac{11}{12} = \mathbf{0.9167}$ as a decimal, correct to 4 decimal places

It is also possible to deal with **mixed numbers** on the calculator.

Press Shift then the $\frac{\square}{\square}$ function and $\square \frac{\square}{\square}$ appears

Problem 13. Evaluate $5\frac{1}{5} - 3\frac{3}{4}$ using a calculator

- (i) Press Shift then the $\frac{\square}{\square}$ function and $\square \frac{\square}{\square}$ appears on the screen
- (ii) Type in 5 then \rightarrow on the cursor key
- (iii) Type in 1 and \downarrow on the cursor key
- (iv) Type in 5 and $5\frac{1}{5}$ appears on the screen
- (v) Press \rightarrow on the cursor key
- (vi) Type in $-$ and then press Shift then the $\frac{\square}{\square}$ function and $5\frac{1}{5} - \square \frac{\square}{\square}$ appears on the screen
- (vii) Type in 3 then \rightarrow on the cursor key
- (viii) Type in 3 and \downarrow on the cursor key
- (ix) Type in 4 and $5\frac{1}{5} - 3\frac{3}{4}$ appears on the screen
- (x) Press = and the answer $\frac{29}{20}$ appears

(xi) Press shift and then S \Leftrightarrow D function and $1\frac{9}{20}$ appears

(xii) Press S \Leftrightarrow D function and the fraction changes to a decimal 1.45

Thus, $5\frac{1}{5} - 3\frac{3}{4} = \frac{29}{20} = 1\frac{9}{20} = 1.45$ as a decimal

Now try the following Practice Exercise

Practice Exercise 3 Fractions (Answers on page 504)

In problems 1 to 3, evaluate the given fractions.

1. $\frac{1}{3} + \frac{1}{4}$ 2. $\frac{1}{5} + \frac{1}{4}$ 3. $\frac{1}{6} + \frac{1}{2} - \frac{1}{5}$

In problems 4 and 5, use a calculator to evaluate the given expressions.

4. $\frac{1}{3} - \frac{3}{4} \times \frac{8}{21}$ 5. $\frac{3}{4} \times \frac{4}{5} - \frac{2}{3} \div \frac{4}{9}$

6. Evaluate $\frac{3}{8} + \frac{5}{6} - \frac{1}{2}$ as a decimal, correct to 4 decimal places.

7. Evaluate $8\frac{8}{9} \div 2\frac{2}{3}$ as a mixed number.

8. Evaluate $3\frac{1}{5} \times 1\frac{1}{3} - 1\frac{7}{10}$ as a decimal, correct to 3 decimal places.

9. Determine $\frac{2}{x} + \frac{3}{y}$ as a single fraction.

1.3 Percentages

Percentages are used to give a common standard. The use of percentages is very common in many aspects of commercial life, as well as in engineering. Interest rates, sale reductions, pay rises, exams and VAT are all examples where percentages are used.

Percentages are fractions having 100 as their denominator.

For example, the fraction $\frac{40}{100}$ is written as 40% and is read as 'forty per cent'.

The easiest way to understand percentages is to go through some worked examples.

Problem 14. Express 0.275 as a percentage

$$0.275 = 0.275 \times 100\% = 27.5\%$$

Problem 15. Express 17.5% as a decimal number

$$17.5\% = \frac{17.5}{100} = 0.175$$

Problem 16. Express $\frac{5}{8}$ as a percentage

$$\frac{5}{8} = \frac{5}{8} \times 100\% = \frac{500}{8}\% = 62.5\%$$

Problem 17. In two successive tests a student gains marks of $\frac{57}{79}$ and $\frac{49}{67}$. Is the second mark better or worse than the first?

$$\frac{57}{79} = \frac{57}{79} \times 100\% = \frac{5700}{79}\% = 72.15\%$$

correct to 2 decimal places.

$$\frac{49}{67} = \frac{49}{67} \times 100\% = \frac{4900}{67}\% = 73.13\%$$

correct to 2 decimal places.

Hence, **the second test mark is marginally better than the first test.**

This question demonstrates how much easier it is to compare two fractions when they are expressed as percentages.

Problem 18. Express 75% as a fraction

$$75\% = \frac{75}{100} = \frac{3}{4}$$

The fraction $\frac{75}{100}$ is reduced to its simplest form by cancelling, i.e. dividing numerator and denominator by 25.

Problem 19. Express 37.5% as a fraction

$$\begin{aligned} 37.5\% &= \frac{37.5}{100} \\ &= \frac{375}{1000} \text{ by multiplying numerator and} \\ &\quad \text{denominator by 10} \\ &= \frac{15}{40} \text{ by dividing numerator and} \\ &\quad \text{denominator by 25} \\ &= \frac{3}{8} \text{ by dividing numerator and} \\ &\quad \text{denominator by 5} \end{aligned}$$

Problem 20. Find 27% of £65

$$27\% \text{ of } £65 = \frac{27}{100} \times 65 = \text{£}17.55 \text{ by calculator}$$

Problem 21. A 160 GB iPod is advertised as costing £190 excluding VAT. If VAT is added at 20%, what will be the total cost of the iPod?

$$\text{VAT} = 20\% \text{ of } £190 = \frac{20}{100} \times 190 = £38$$

$$\text{Total cost of iPod} = £190 + £38 = \text{£}228$$

A quicker method to determine the total cost is:
 $1.20 \times £190 = \text{£}228$

Problem 22. Express 23 cm as a percentage of 72 cm, correct to the nearest 1%

$$\begin{aligned} 23 \text{ cm as a percentage of } 72 \text{ cm} &= \frac{23}{72} \times 100\% \\ &= 31.9444\dots\% \\ &= \text{32\% correct to the} \\ &\quad \text{nearest 1\%} \end{aligned}$$

Problem 23. A box of screws increases in price from £45 to £52. Calculate the percentage change in cost, correct to 3 significant figures.

$$\begin{aligned} \% \text{ change} &= \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\% \\ &= \frac{52 - 45}{45} \times 100\% = \frac{7}{45} \times 100 \\ &= \text{15.6\%} = \text{percentage change in cost} \end{aligned}$$

Problem 24. A drilling speed should be set to 400 rev/min. The nearest speed available on the machine is 412 rev/min. Calculate the percentage over-speed.

$$\begin{aligned} \% \text{ over-speed} &= \frac{\text{available speed} - \text{correct speed}}{\text{correct speed}} \times 100\% \\ &= \frac{412 - 400}{400} \times 100\% = \frac{12}{400} \times 100\% = \text{3\%} \end{aligned}$$

Now try the following Practice Exercise

Practice Exercise 4 Percentages (Answers on page 504)

In problems 1 and 2, express the given numbers as percentages.

- 0.057
- 0.374
- Express 20% as a decimal number
- Express $\frac{11}{16}$ as a percentage
- Express $\frac{5}{13}$ as a percentage, correct to 3 decimal places
- Place the following in order of size, the smallest first, expressing each as percentages, correct to 1 decimal place: (a) $\frac{12}{21}$ (b) $\frac{9}{17}$ (c) $\frac{5}{9}$ (d) $\frac{6}{11}$
- Express 65% as a fraction in its simplest form
- Calculate 43.6% of 50 kg
- Determine 36% of 27 m
- Calculate correct to 4 significant figures: (a) 18% of 2758 tonnes (b) 47% of 18.42 grams (c) 147% of 14.1 seconds
- Express: (a) 140 kg as a percentage of 1 t (b) 47 s as a percentage of 5 min (c) 13.4 cm as a percentage of 2.5 m
- A computer is advertised on the internet at £520, exclusive of VAT. If VAT is payable at 20%, what is the total cost of the computer?
- Express 325 mm as a percentage of 867 mm, correct to 2 decimal places.
- When signing a new contract, a Premiership footballer's pay increases from £15,500 to £21,500 per week. Calculate the percentage pay increase, correct 3 significant figures.
- A metal rod 1.80 m long is heated and its length expands by 48.6 mm. Calculate the percentage increase in length.